Total No. of Printed Pages:05

SUBJECT CODE NO:- H-111
FACULTY OF ENGINEERING AND TECHNOLOGY
S.E. (All Branches)
Engineering Mathematics - IV
(REVISED)

[Time: Three Hours] **[Max. Marks: 80]**

Please check whether you have got the right question paper.

N.B
1. Q. No. 1 and 6 are compulsory
2. Solve any two questions from the remaining questions of each section
3. Figures to the right indicate full marks
4. Assume suitable data, if necessary

Section A

Q.1 Attempt any five 10

1. Find the Laplace transform of $sin2t\ sin3t$
2. Find Laplace transform of $t^2H\ (t-2)$
3. Find Laplace transform of $f(t) = (t-2)^2, t > 2$
   $= 0, \quad t < 2$
4. Find the inverse Laplace transform of $\frac{S+2}{S^2-4S+13}$
5. Find the inverse Laplace transform of $\frac{S\,e^{-3s}}{S^2-1}$
6. Form the partial differential equation from

   $$(x-h)^2 + (y-k)^2 = a^2 - z^2$$

   OR

   Find Z-transform of $\sin h\left(\frac{k\pi}{2}\right), k \geq 0$
7. Solve: $pq = p + q$

   OR

   Find the z-transform of $ke^{ak}, K \geq 0$

8. Solve $x\frac{\partial z}{\partial x} - 4y\ \frac{\partial z}{\partial y} = 0$

OR

Find the z-transform of $2^k cos\ h \propto k,\ k \geq 0$

Q.2

a. Find the Laplace transform of $\int_0^t t\cos^2 t\ dt$ 05

b. Find the inverse Laplace transform of $\tan^{-1}(s)$ 05

c. Solve: $p^2 + q^2 = Z$ 05

OR

Find z-transform of $\cos(\frac{k\pi}{3} + 5)$

Q.3

a. Evaluate : $\int_0^\infty e^t\ \frac{\sin^2 t}{t}\ dt$ 05

b. Find the inverse Laplace transform by using convolution theorem 05

$$\frac{s}{s^4+8s^2+16}$$

c. Solve $x^2p + y^2q + z^2 = 0$ 05

OR

Find the inverse z- transform of $\frac{z}{(z-2)(z+3)^2}$, $|z| > 3$

Q.4

a. Find the Laplace transform of periodic function 05

$$f(t) = E, 0 < t < \frac{p}{2}$$
$$= -E, \quad \frac{p}{2} < t < p$$

And $f(t) = f(t+p)$

b. Solve by Laplace transform $y'' + 2y' + 5y = e^{-t}sint$; given that $y(0) = 0, y'(0) = 1$ 05

c. Obtain the solution of $\frac{\partial v}{\partial t} = k\frac{\partial^2 v}{\partial x^2}$ under the conditions 05

i. $v \neq \infty, t \to \infty$
ii. $v = 0, for\ x = 0\ and\ x = \pi$
iii. $u = \pi x - x^2\ as\ \ t = 0$ in the range of $(0, \pi)$

OR

Find the z-transform of $k^2, k \geq 0$

Q.5 a. Express the following function in terms of Heaviside unit step function and hence find its Laplace transform $f(t) = (t+1), 0 < t < 2$
$= 3, \quad t > 2$ 05

b. Solve the simultaneous L.D.E. by Laplace transform method 05

$\frac{dx}{dt} + 4y = 0, \frac{dy}{dt} - 9x = 0$, given

$x = 2, y = 1 \; at \; t = 0$

c. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{d^2 u}{\partial y^2} = 0$ which satisfies the conditions $u(0, y) = 0 = u(l, y)$ 05
And $u(x, \infty) = 0, \quad u(x, 0) = kx$

OR

Solve the difference equation by using z- transform
$6y(k+2) - y(k+1) - y(k) = 0, k \geq 0$
Given y(0)= y(1) =1

Section B

Q.6 Attempt any five 10

a. Prepare a forward difference table for the data

x :	2	4	6	8	10
y :	15	23	27	33	40

b. Find the first approximate value of the root (i.e. x_1) by Newton Raphson method for $\log x - x + 3 = 0$

c. Find the values of x, y, z in the first iteration of Gauss –Seidal method
$83x + 11y - 4z = 95$
$7x + 52y + 13z = 104$
$3x + 8y + 29z = 71$

d. Find f(8) for the data

x :	5	6	9
f(x) :	12	13	14

e. Find the residues at each of its poles of
f(z) = $\frac{3z^2}{(z-1)(z+3)}$

f. Evaluate $\int_c sinhz\ dz$, where c : |z| =1

g. Show that coshz is analytic every where

h. State Cauchy-Riemann equations in Cartesian and polar form

Q.7 a. Given that $\frac{dy}{dx} = 2 + \sqrt{xy}$ and y (1)=1. Find the approximate value of y at x=1.2 using Euler's modified method 05

b. Use runge-kutta fourth order method to find y at x=0.2. given that 05

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \quad , \quad y(0) = 1$$

c. Under the transformation $w = \frac{1}{z}$ find the image of $x^2 + y^2 - 6x = 0$ 05

Q.8 a. Find the root of the equation $x\ log_{10}^{x} = 1.2$ by Newton-Raphson method 05

b. Prove that $u = r^3 \cos 3\theta - rsin\theta$ is harmonic and hence find its harmonic conjugate 05

c. Evaluate by cauchy's Residue Theorem 05

$\oint_c \frac{z^2}{(z-1)(z+2)^2} dz$, c: |z|= $\frac{3}{2}$

Q.9 a. State cauchy's integral formula and hence evaluate 05

$\oint_c \frac{z+1}{(z^3-4z)} dz$, where c: $|Z + 2| = \frac{3}{2}$

b. Find the bilinear transformation which maps the points -1, 0, 1 into the points $-1, -i, i$ of w-plane respectively 05

c. Fit a second degree parabola to the following data 05

x :	0	1	2	3	4
y :	-4	-1	4	11	20

Q.10 a. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x=0 for the data 05

x :	0	1	2	3	4
y :	2	5	10	14	19

b. Find the analytic function $f(z) = u + iv \ \ if \ v = \left(r - \frac{1}{r}\right) sin\theta$ 05

c. Evaluate $\int_0^{\pi - \pi i} e^{\bar{z}} dz$, along the curve $x = t, y = -t$ 05